Transient Perfect Second Order 2-Way Crossover Implemented Using Passive Networks*

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Abstract

A set of design rules is developed for the construction of transient perfect, 2nd order, symmetric crossovers for 2-way speaker systems. The designs employ high-pass and low-pass filter sections with standard, passive topology. Overlap of the filter sections is use to obtain a minimum phase, but not flat response for the summed filter output. A simple equalization circuit, which can also be passively implemented, is introduced to provide the necessary correction to obtain flat response. Optimum relationships between the degree of overlap, filter section characteristics and the required equalization are developed through computer simulation. It is shown that the resulting attenuation rated always lie between the limits of standard 1st order Butterworth and 2nd Linkwitz-Riley crossovers. A simple resistively loaded test crossover is built and tested to demonstrate the results experimentally. Finally, application to the design of real loudspeakers is discussed.

Introduction

The ideal loudspeaker should produce as its output a time dependent acoustic waveform that is identical to the input electrical waveform. Theoretically, such a loudspeaker would necessarily posses flat, infinite bandwidth and would be either a minimum phase device or introduce a phase shift corresponding to a constant time delay. That is to say, the speaker would be transient perfect. Due to the limited bandwidth of sound reproducing devices, be they dynamic drivers, planar magnetic or electrostatic panels, or ribbon devices, this ideal speaker is unattainable. The closest approximation is that system which has the broadest range of flat frequency response with either of the phase characteristics given above. Unfortunately, with very few exceptions, the best that today's technology has to offer from single drivers is a response that extends over 7 or, in extreme cases, 8 of the 10 octave audio band, 20 Hz to 20k Hz. When dealing with conventional dynamic drivers this range is typically more limited. For example, high quality, 28mm dome tweeters are limited, at best, to a useful range extending over the top 4 octaves (1k Hz to 20k Hz.). Similarly, a high quality, 13cm mid woofer is limited to a useful range of about 6 octaves (50 to 3.5k Hz). Thus, speaker designers are forced to compromise and employ multiple drivers to cover the audio band. This use of multiple drivers necessitates the insertion of some kind of electrical network to blend the acoustic output of the drivers into a coherent sound field. When the radiation characteristics of the drivers employed and the interference patterns that arise due to the use of multiple drivers is taken into consideration, it quickly becomes apparent that the ideal response desired will be achievable over only a very limited position in space. However, this should not a deterrent from pursuing the ideal of a transient perfect design.

A Brief Review of Previous Work

It is a common misconception that the desired transient perfect response is obtainable only with a first order crossover using a Butterworth alignment. However, Small [1] showed that transfer functions for so called constant voltage crossovers, with nonsymmetrical high-pass (HP) and low-pass (LP) filter sections, that have flat summed amplitude and phase response, are easily developed. Given that $G_L(s)$ and $G_H(s)$ are the transfer functions of the LP and HP sections, respectively, then the desired result way be expressed as,

$$G_L(s) + G_H(s) = 1. \quad (1)$$

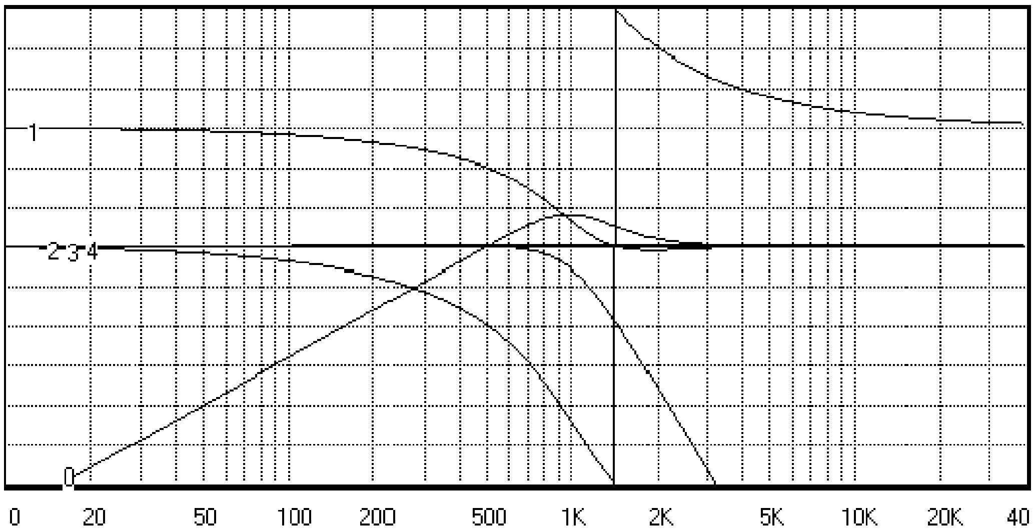

Figure 1. Constant voltage crossover HP and LP sections employing a 3rd order Butterworth low-pass filter. 0 - HP amplitude response, 1-HP phase response. Scale: 5db/division

Eq. (1) is a vector equation involving both amplitude and phase. It follows that for any choice of HP or LP section, the complimentary filter section can be obtained from Eq. (1). For example, choosing the filter characteristic for the HP section, the LP section transfer function is given by,

$$G_L(s) = 1 - G_H(s). \quad (2)$$

The problem with using this approach is that while the summed response of the HP and LP filter sections yields a transient perfect response, the transfer function of the complementary filter ultimately has a 6db per octave roll off [1]. Additionally, Small [1] indicated that, other than in the 1st order case, these constant voltage crossovers can not be implemented using driver terminated, pass networks.

Figure 1[1] shows the HP and LP filter section response curves derived from Eq. 2 when a 1k Hz, 3rd order Butterworth filter is selected for the low-pass section. The decibel scale is 5 db per division in the figure. Observe that the HP response has a peak of 4 db at the LP -3 db point and that the HP section is down 3 db at approximately 350 Hz. A similar result, when the HP filter is chosen to be a 1k Hz, 3rd order Butterworth filter, is shown in figure 2. In both cases, and all case to follow, both filter sections are connected with normal polarity.

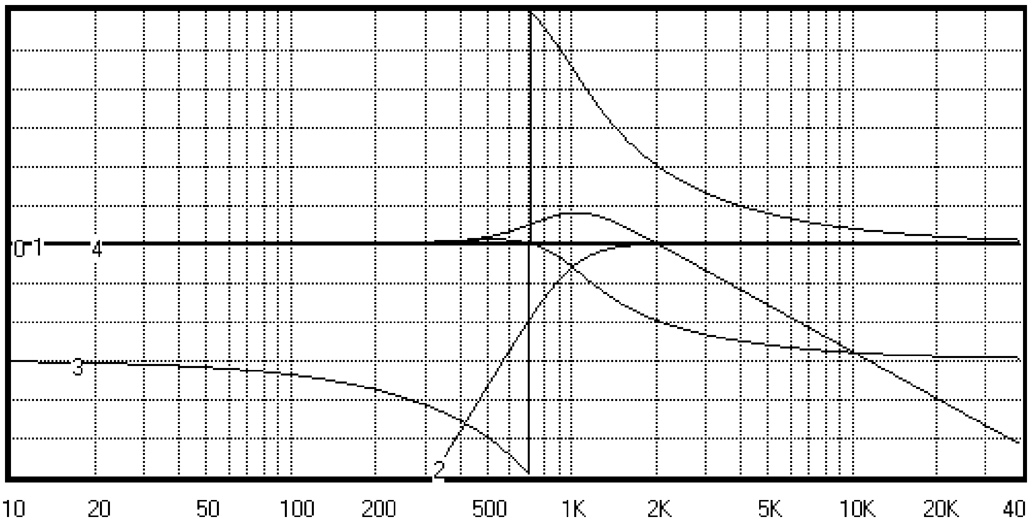

Figure 2. Constant voltage crossover HP and LP sections employing a 3rd order Butterworth high-pass filter. Scale: 5db/division

Small [1] also showed that it was possible to derive constant voltage crossovers with symmetric filter characteristics. He provided an example using filters with a 2nd and 3rd order asymptotic slopes. Again, these filters have unconventional transfer functions and are not suited for implementation using passive networks. Some 14 years later, Lipshitz and Vanderkooy [2] revisited the use of higher order filter functions, and in particular symmetric filter functions, in an effort to produce linear phase, high slope crossover networks. Contrary to Small, Lipshitz and Vanderkooy examined standard higher order filter alignment including the Butterworth, Bessel, and Linkwitz-Riley (L-R). They showed that by overlapping the HP and LP sections of these symmetric filters, the summed response could be reduced to a minimum phase response. However, the amplitude response of these overlapped filters was not flat. The object was to then equalize the response using an additional minimum phase circuit with the net result of a flat, minimum phase response. Mathematically this may be expressed as

$$\boldsymbol{G_{eq}(s) \times (G_L(s) + G_H(s)) = 1.} \qquad (3)$$

[1] Note: The majority of figures appearing in this article were created using Sound Easy by Bodzio Software, Aus.

and the required equalization response is then given as

$$Geq(s) = 1. / (G_L(s) + G_H(s)) \tag{4}$$

where ***G**eq**(s)*** is the transfer function of the equalization network. Lipshitz and Vanderkooy concluded that crossovers employing filters of higher than 3rd order required *unreasonable* equalization and were therefore of limited use. They also provided the criteria for the minimum overlap required to render the summed response minimum phase, but they did not indicate how equalization circuits for any order crossover could be developed.

Following Lipshitz and Vanderkooy, for a given crossover frequency, ***fo***, the corner frequencies of the HP and LP sections are related to the overlap parameter γ as,

$$Fh = fo \ / \sqrt{\gamma} \tag{5}$$

$$Fl = fo \times \sqrt{\gamma} \tag{6}$$

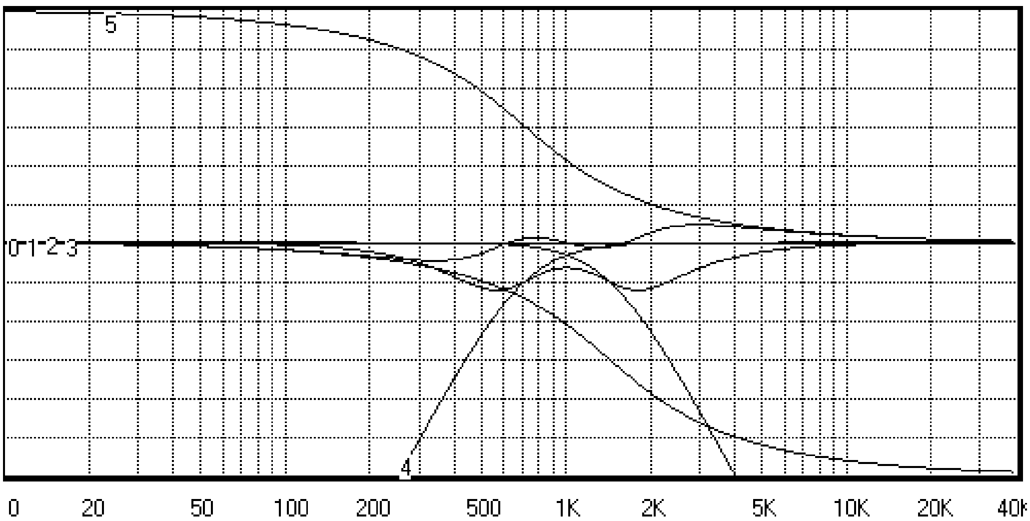

Figure 3. Summed 2nd order overlapped Butterworth crossover at 1k Hz following Lipshitz and Vanderkooy [2]. Overlap parameter = 2. Scale: 3db/division

Based on the relationships of Eqs. 5 & 6, figure 3 shows the summed amplitude and phase response for 1K Hz crossover derived from second order Butterworth HP and LP sections with an overlap parameter of 2. Note the peak in the amplitude response at the crossover frequency, 1k Hz, and the dips to either side. These dips give rise to multiple inflection points before the response goes to flat band below 100 Hz and above 10k Hz. Also note the phase response, which is indicative of a minimum phase system in accordance with Bode's theorem [3]. It also exhibits a double crested behavior, with multiple inflection points. In contrast, figure 4 shows the result when 2nd order Linkwitz-Riley filter sections are used in place of the Butterworth sections. With these filter sections the summed amplitude response is smooth with a single dip at the 1K Hz crossover point. The phase response also exhibits a smooth functional shape with a single inflexion point at the crossover frequency. As will become apparent, this observation proves to be an important one. Finally, the result using 3rd order Butterworth filter sections with an overlap parameter of 4 is shown in figure 5. The response resembles that of the 2nd order Butterworth system but with much deeper dips in the amplitude response and more significant phase variation. However, the phase response is still minimum phase.

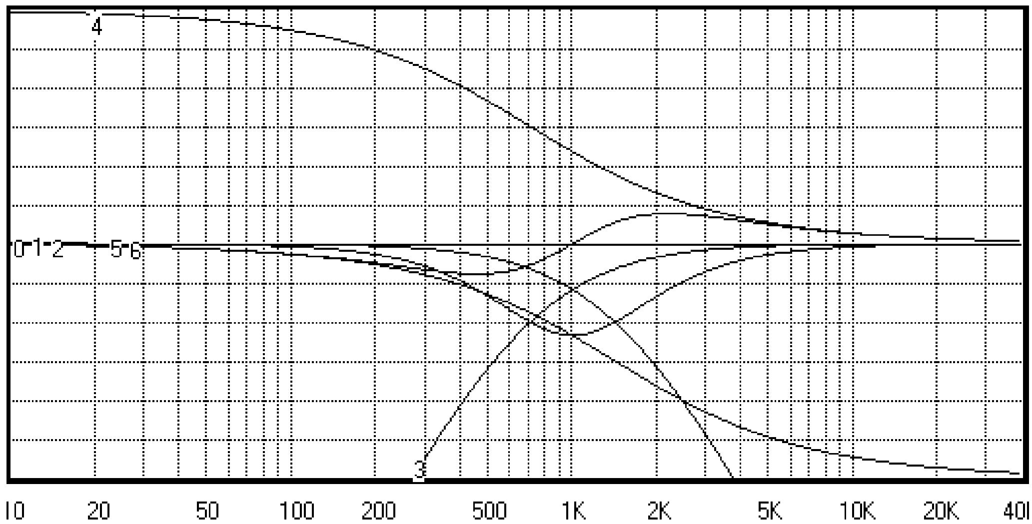

Figure 4. Summed 2nd order overlapped Linkwitz-Riley crossover at 1k Hz following Lipshitz and Vanderkooy [2]. Overlap parameter = 2. Scale: 3db/division

The Present Approach

The approach to developing a transient perfect crossover that I have followed is an extension of the work of Lipshitz and Vanderkooy [2], limited to the use of 2nd order filters. However, rather than require the filter transfer functions be those of traditional filters, the specific alignments of the HP and LP filters were considered to be variables in the overall design. The alignment is to be determined, in conjunction with the alignment of a specified equalization circuit, in an effort to achieve flat amplitude and phase response of the final system. As will become apparent, a class of

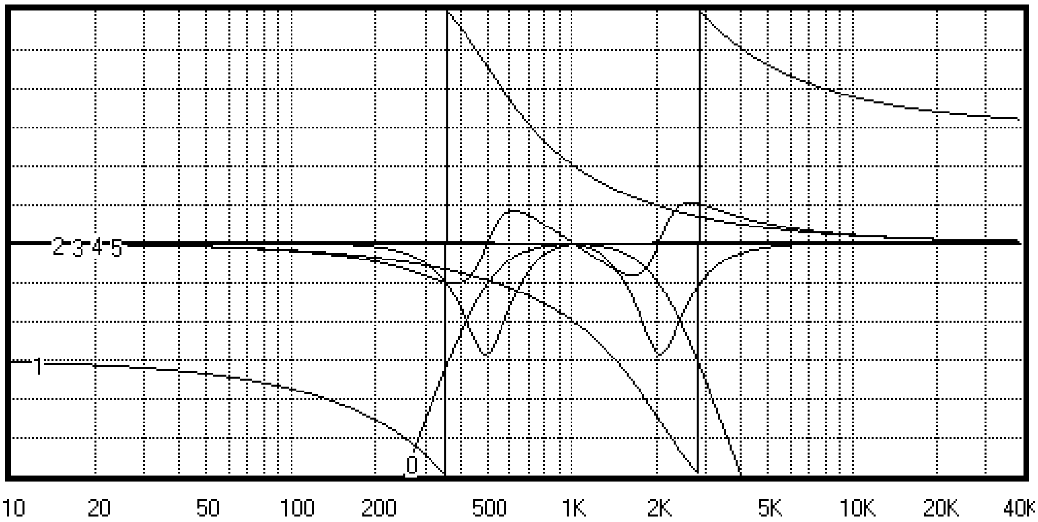

Figure 5. Summed 3rd order overlapped Butterworth crossover at 1k Hz following Lipshitz and Vanderkooy [2]. Overlap parameter = 4. Scale: 3db/division.

equalized, 2nd order crossover networks for which the overlap, and filter and equalization section Qs are optimum for the required level of equalization was found. The filters were developed with the intent that the equalization be implemented through the use of passive circuitry. However, an active equalization circuit with the same transfer function could be substituted.

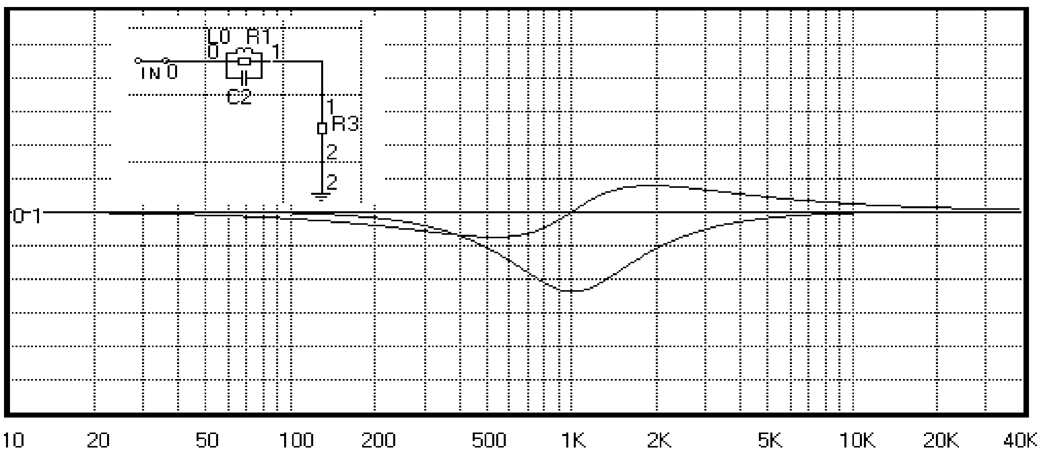

Figure 6.Trap circuit that has similar response to overlapped L-R crossover. Scale: 3db/division.

Recall the summed response of the Linkwitz-Riley overlapped crossover shown in figure 4. It was observed that this response looked very similar to that of an RLC trap circuit. Such a circuit and its response are shown in figure 6. This similarity lead to consideration of a passive equalization circuit as shown in figure 7, where R_4 represents the load due to the crossover. Using the Linkwitz-Riley HP and LP filter sections with the γ=2, the required equalization is in excess of 6 db which may be considered excessive. However, to determine if the approach was feasible, the circuit shown in figure 8 was set up in CALSOD [4] and the components for the equalization circuit were optimized. The target response level was set at the minimum in the unequalized response, which occurs at the crossover frequency (see figure 4). The filter functions were held fixed at the Linkwitz-Riley values and the optimization performed. The results of the optimization were encouraging. The frequency response was flat to within ±0.25 db with the corresponding minimum phase response.

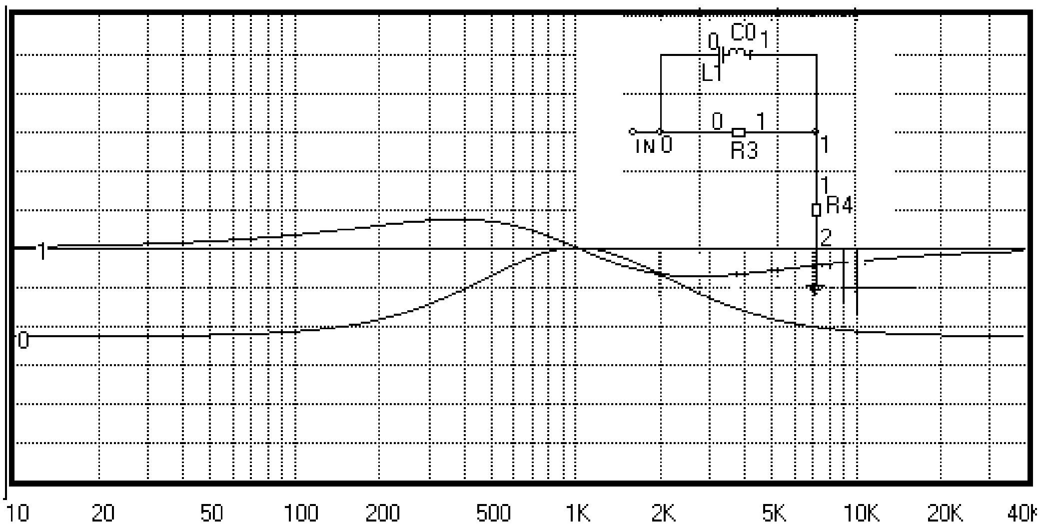

Figure 7. Passive equalization circuit for overlapped 2nd order Linkwitz-Riley crossover. Scale: 3db/division.

The lack of achieving a perfectly flat response indicated that the shape of the response of the summed, unequalized filters did not exactly correspond to that obtainable the resonant RLC equalization network. That is to say, ***Geq(s),*** as would be determined from Eq. (4), did not correspond to a functional form obtainable from an RLC circuit. To determine if a more accurate response could be obtained with the simple RLC equalization circuit, the optimization was repeated with all circuit variables included in the optimization procedure. This was successful and yielded a perfectly flat amplitude response with zero phase shift; the perfect second order, symmetric, passively implemented crossover. Analysis of the circuit components showed that the overlap had changed somewhat as did the filter section *Q*. However, the only significant negative factor was the rather excessive power dissipation which occurred in what would normally be considered the flat band, or pass band regions of the unequalized HP and LP filter sections. The observation that slightly altering the filter section *Q* and overlap allowed for a perfectly flat summed, equalized response also suggested to me that it might be possible to determine specific values of the filter *Q* and overlap for different levels of attenuation. The attenuation corresponding to the depth of the dip in the summed, unequalized amplitude response at the crossover frequency. Determination of the relationships between this attenuation and the filter section Q_f, the overlap, γ, and Q_{eq} of the equalization circuit is considered next.

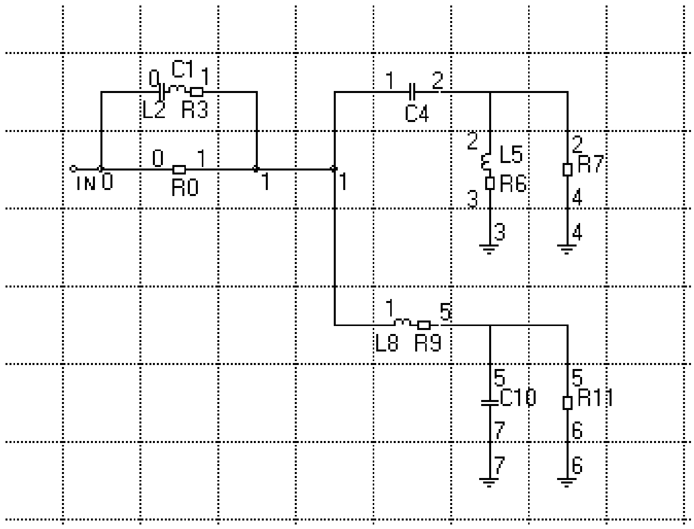

Figure 8. Second order crossover and equalization used in CALSOD optimization.

Excessive Attenuation

The problem of the excessive power dissipation, or attenuation, required to passively equalize the crossover was considered next. Since the equalization circuit is rather simple, it would not be difficult to construct and active equalizer to be placed between the preamplifier and amplifier. Such a system would have the added benefits that it would ultimately allow for easy biwiring or biamplifying, and the HP and LP sections would then default to relative standard type filter functions closely approximating the Linkwitz-Riley characteristic. However, the use of active equalization may be beyond some do it yourselfers, or just deemed undesirable in a speaker design. In any case, it was desired to keep the system passive. It was felt that there would clearly be a benefit to a purely passive system, with active equalization viewed as an available option. In view of the fact that many of today's commercial speaker offerings operate in the range of 82 to 90 db/w/m, (or perhaps better stated as 82 to 90 db/ 2.83 volts/m), it would seem that if the equalization could be brought into the 4 or 5 db range the passive approach would be acceptable. With today's higher quality, higher sensitivity drivers, it was anticipated that systems with sensitivities in the 84 to 86 db/w/m could be constructed. While perhaps not the most efficient, this is considered this to be a reasonable tradeoff.

In an effort to determine if such systems could be developed with passive equalization a series of calculations was made with CALSOD setting different target attenuation levels for the equalization circuit. The results were successful and quite interesting. In every case the optimization procedure found solutions for symmetric HP and LP filter functions which yielded perfectly flat response and zero phase shift to within the limits set for convergence. The results are summarized in figures 9-11 where the filter section Q_f, the overlap parameter, γ and the Q of the required

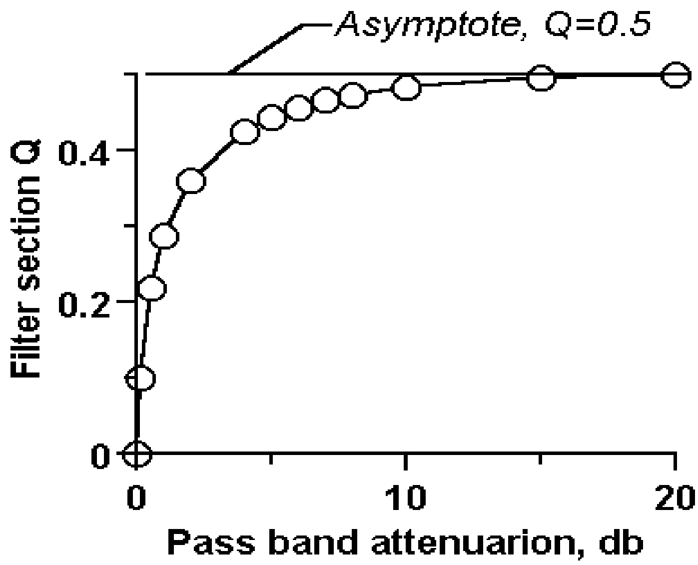

Figure 9. Filter section Q vs. pass band attenuation.

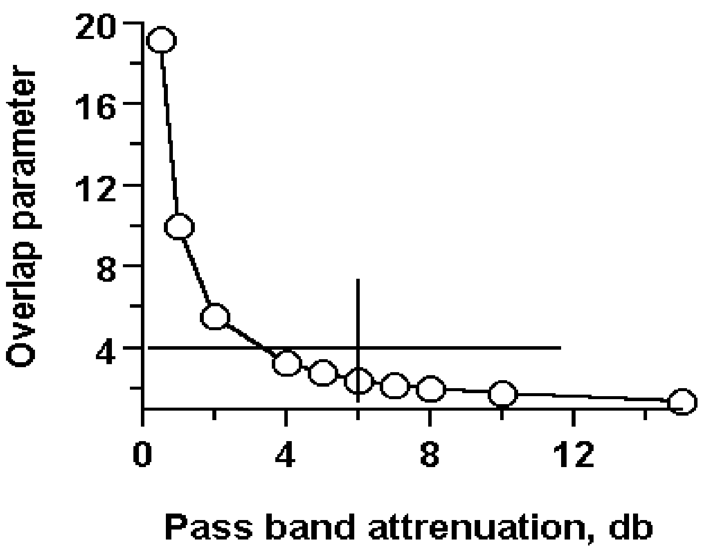

Figure 10. Overlap parameter vs. pass band attenuation

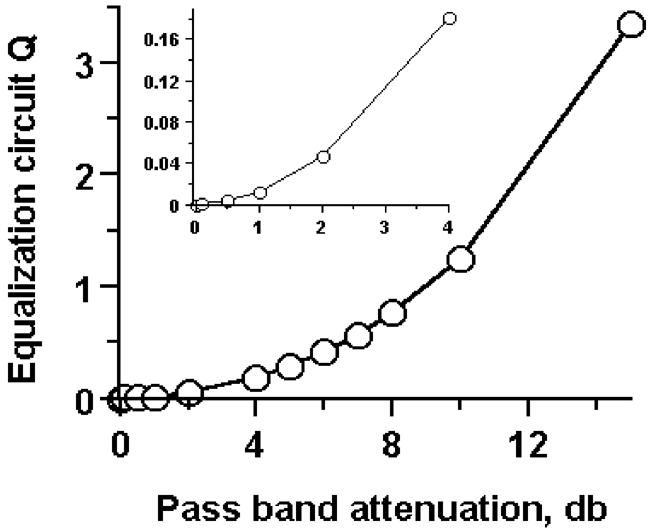

Figure 11. Q of required equalization circuit.

equalization circuit, $\boldsymbol{Q_{eq}}$, as function of the required pass band attenuation are show. In the extreme, as the attenuation goes to 0 db, the filter section *Q* goes to zero and the overlap parameter goes to infinity. The interpretation of this is that the filters reduce to the standard 1st order Butterworth alignment with a crossover frequency of ***fo*** at this point. At the other extreme, as the attenuation approaches infinity, the filter section *Q* goes to 0.5 and the overlap parameter is *unity*. This is the standard Linkwitz-Riley crossover with the drivers wired in phase, and is the limiting minimum phase configuration [2]. An equalization circuit with a *Q* of infinity and infinite flat band attenuation would render this limiting case theoretically flat. However, a minimum phase crossover with a flat response of *zero* amplitude is of no practical use. Yet it is an important result since it shows that filter sections with *Q* greater that 0.5 can not be overlapped and equalized in a manner that will yield flat response with a simple second order RLC equalization circuit.

In addition to the loss of power when using passive equalization, excessive equalization used with low overlap should be avoided due to the excessive and steep phase variations that appear in the *individual* filter responses under such conditions. While the summed system response is theoretically flat on the design axis in any case, such rapid phase variations will adversely affect the off axis response of a multiple driver system. Similarly, excessive overlap defeats the intent of the design: that of limiting the operational bandwidth of the HP and LP sections relative to that of a 1st order system. It is recommendation that acceptable solutions are in the area of 6 db attenuation or less, with an overlap parameter of less than 4. The boxed region in figure 10 highlights this range. As an example, at 4 db attenuation the filter section *Q* is approximately 0.425, the overlap parameter about 3.25 and the *Q* of the required equalization circuit is approximately 0.18. This represents a reasonable design point. Choosing a cross over frequency of 1k Hz, the HP and LP corner frequencies are giver by Eqs. (5) and (6) as,

$$F_H = 1000 / \sqrt{\gamma} \approx 555 \text{ Hz} \quad (7)$$

$$F_L = 1000 \times \sqrt{\gamma} \approx 1803 \text{ Hz} \quad (8)$$

The inductor and capacitor values for the filter sections are obtained for the filter Q_f and the load resistance ***R,***

$$C_f = Q_f / (2\pi f \times R) \quad (9)$$

$$L_f = R / (2\pi f \times Q_f) \quad (10)$$

and the required resistance for the equalization circuit is given by,

$$R_{eq} = R\,(1 - Log^{-1}[-A/20]\,) / Log^{-1}[-A/20] \quad (11)$$

where ***R*** is again the load resistance and ***A*** is the desired attenuation, 4 db in this case. Finally, the inductor and capacitor values for the equalization circuit are given as,

$$C_{eq} = 1/Q_f\,(2\pi fo \times R) \quad (12)$$

$$Leq = R \times Q_f / 2\pi fo \quad (13)$$

Here, ***R*** is, once again, the load resistance and ***fo*** is the crossover frequency.

A simulation was performed of a crossover based on the selected crossover frequency of 1K Hz and a 4 db attenuation with the circuit components given by Eqs. (9-13). The results of this simulation are shown in figures 12 to14. Figure 12 shows the simulated HP and LP filter amplitude response with and without equalization, the summed response, with and without equalization, and the response of the equalization circuit alone. Examination of the figure shows the smooth roll off of the unequalized HP and LP sections, typical of a second order filter with a Q near the critically damped value. Also note that the equalized filter responses show peaks of approximately 2 db at the corner frequencies for the HP and LP sections, respectively. Finally, note the symmetry of the equalizer response and the summed, unequalized filter response about the -2 db point.

The phase response of the unequalized HP and LP sections, along with the phase response of the summed, unequalized response and the phase of the equalization circuit are presented in figure 13. Note that due to the overlap, the phase difference between the HP and LP sections at the

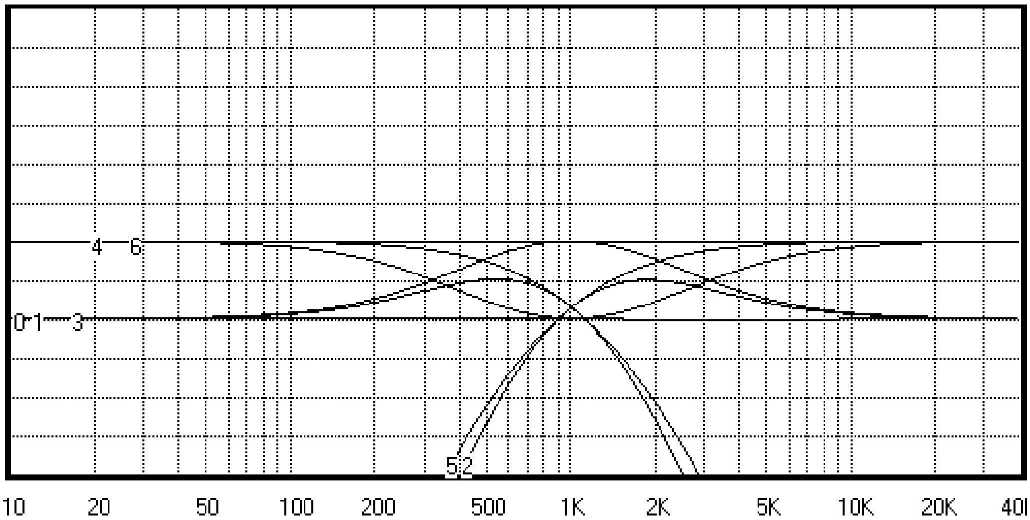

Figure 12. Amplitude response of HP and LP filter sections with and without equalization, summed response with and without equalization, and the equalization response. Scale: 2 db/division.

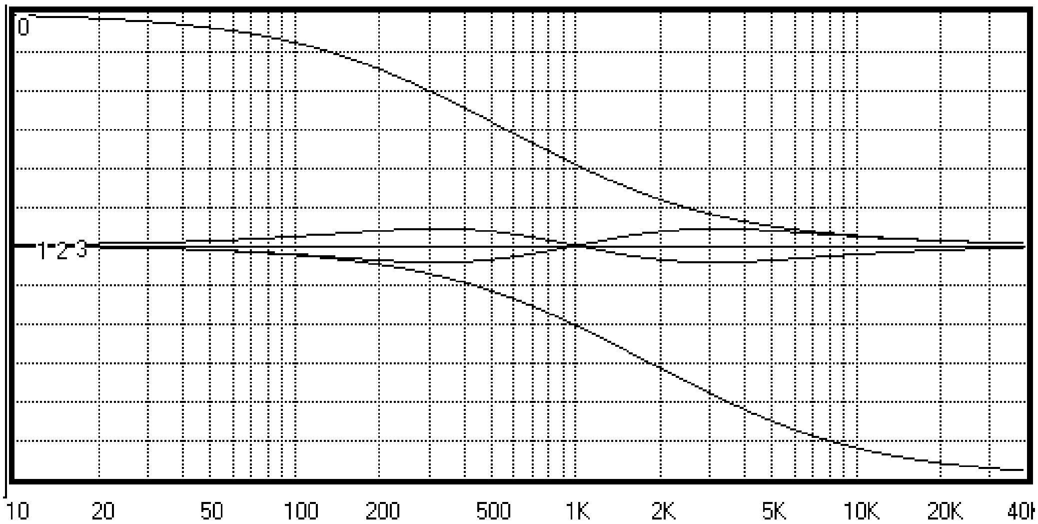

Figure 13. Phase response of unequalized HP and LP sections, summed unequalized phase, and phase of equalization circuit. Scale: 30o per division.

crossover point in approximately 120 degrees as opposed to the typical 180 degrees of an un-overlapped second order crossover. Additionally, the phase of the summed response shows only a little more than ±15 degrees phase shift prior to the application of the equalization. As expected, the phase response of the equalization circuit is symmetric to the summed HP+LP response about the zero axes.

Finally, the simulated response to a 500 Hz square wave is shown in figure 14. The 500 Hz frequency places the fundamental frequency of the square wave below the crossover point with the harmonics above it. As expected for a filter with no phase shift and flat amplitude response, the reproduction is prefect, with the exception of the reduction in amplitude from the application of passive equalization. The time scale in the figure in msec.

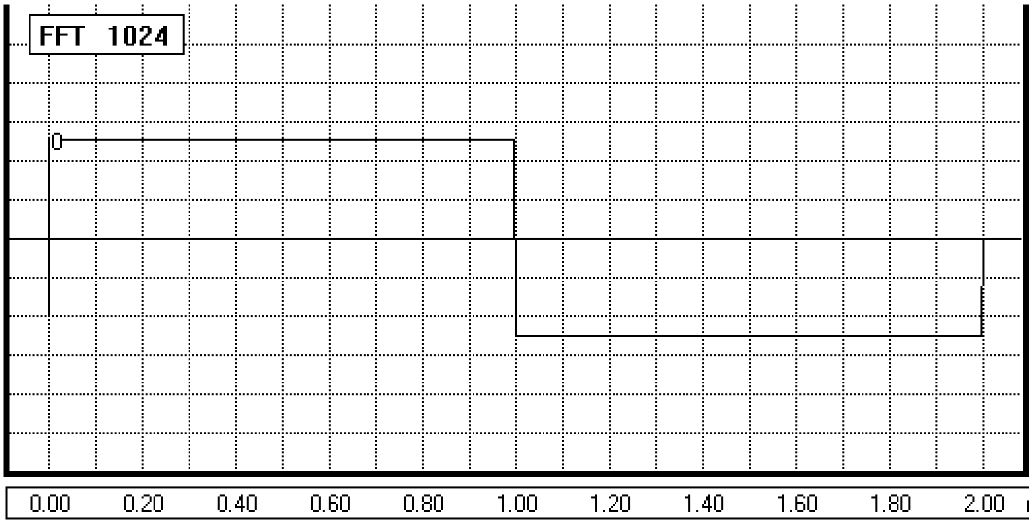

Figure 14. Simulated response to a 500 Hz square wave input.

A Word about Overlap

One of the concerns of this type of crossover or any crossover in which the HP and LP sections are overlapped, when used in the design of a loudspeaker, is that such a filter requires the use of drivers which have extended bandwidth. The question arises, just how much overlap is

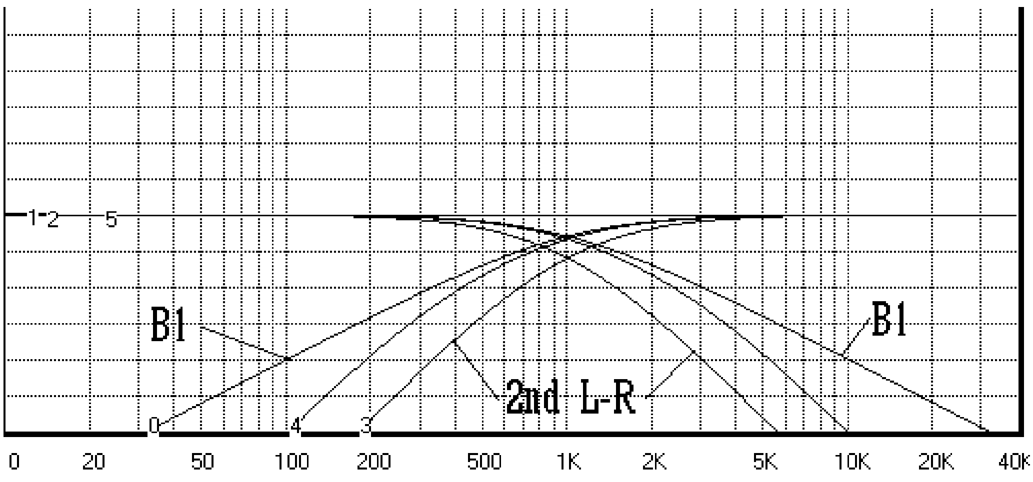

Figure 15. Comparison of 2nd order L-R, 1st order Butterworth and present HP and LP filter amplitude responses. Scale: 5 db/division.

required? Perhaps the easiest way to answer this question is by way of example. In figure 15 the responses of the unequalized HP and LP sections used in the example above are compared with the conventional L-R 2nd order filters and with the standard 1st order Butterworth filters. Recall that for the current class of filters, in the limit of infinite overlap, the crossover reduces to the 1st order Butterworth crossover and the L-R2 filter represents the limit of minimum overlap. Thus, it is apparent from figure 15 that, regardless of the degree of overlap, the roll off of the HP and LP sections of the current crossover will always lie between that of the 1st order Butterworth and the 2nd order L-R crossover.

Vertical Polar Response

Another issue with overlapped higher order crossovers is the effect of the overlap on the vertical polar response. This was investigated by examining the vertical polar response of a theoretical two-way system, using Sound Easy [5], with the woofer tweeter spacing set at 12 cm. The result is compared to that for geometrically similar systems employing 1st order Butterworth and 2nd order L-R crossovers in figure 16. The crossover frequency for the systems is 1k Hz. The polar response was computed as close as possible to this frequency as the software would allow, 988 Hz. It is noted that the polar response for the first order and the present overlapped crossover a quite similar. This is to be expected since, as can be seen in figure 15, in the crossover region the present crossover design follows the 1st order Butterworth response very closely. The L-R crossover exhibits its classical symmetric response. While there has been, and continues to be opposition to the use of 1st order crossovers based on the polar response characteristics, a well thought-out MTM design can go a long way towards alleviating the polar response problems.

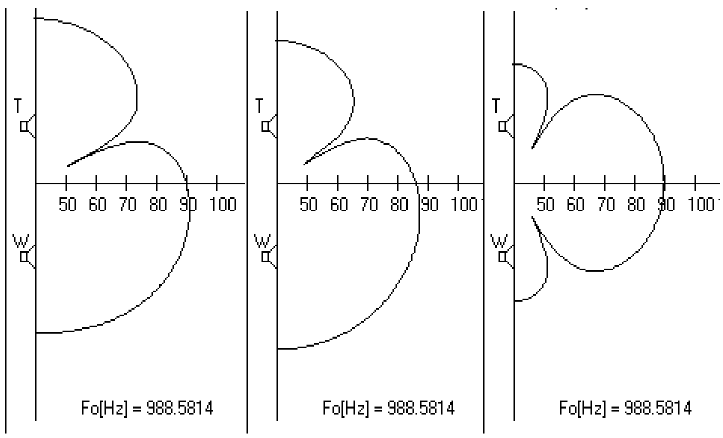

Figure 16. Theoretical vertical polar response at 1K Hz for 1st order Butterworth (left), the present 2nd order overlapped (center) and 2nd order L-R (right) crossovers.

A Quick Experiment

To provide experimental verification that such results can be obtained in practice a 1K Hz crossover was build on a breadboard and tested it. No attempt was made to exactly tune the network, as parts bin parts were used. Figure 17 shows the input 500 Hz square wave, the output of the HP and the LP sections, and the summed response as measure using the IMP [6] as the data acquiring system. Please note that the response curves are *not* synchronized; that is the cycle does not start at the same point in each curve. Never the less, it is clearly evident that theory and practice are in close agreement. The summed output is an excellent representation of the input.

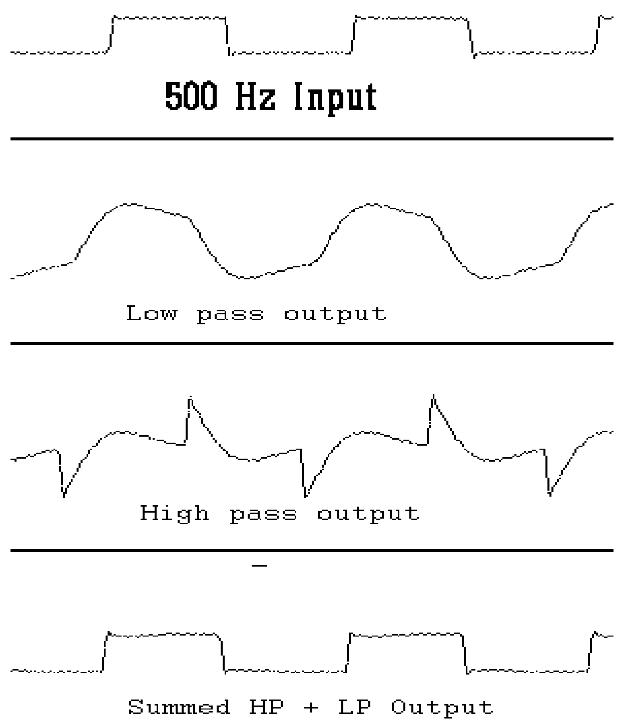

Figure 17. Experimental result for a 500 Hz square wave

Application to Real Loudspeakers

Application of the present crossover design approach to real loudspeakers is complicated by several issues. The facts that real drivers mounted in a box or on a baffle a) have complex, frequency dependent impedance functions, and b) do not necessarily have flat amplitude response are easily addressed. Additionally, it is unlikely that both the woofer and the tweeter would be of the same impedance. This too can be over come fairly directly. However, the low frequency roll off of the woofer/box alignment introduces phase shifts below 100 or 200 Hz, which can not be readily dealt with using passive networks. Accepting this limitation, the first step in the design procedure is choosing drivers which have a suitable overlap in useful frequency range. Once suitable drivers have been specified and the overlap set the required filter section and equalizer Q values are determined from the design curves given in figures 9 and 11. The three components necessary, the HP, LP and equalization section, are all straightforward and simple 2nd order systems. Just as in the case of the design of a 2nd L-R crossover, the HP and LP sections must be designed to yield acoustic response functions that match that of the specific filter function. That is to say, it is the acoustic output of the drivers, measured at the design point that must match the amplitude and phase response of the corresponding filter transfer function, and not the electrical signal applied at the driver terminals. Provided that the networks employed in developing the acoustic filter response are minimum phase networks, the acoustic response of the driver plus the filter will also be minimum phase owing to the minimum phase nature of loudspeaker drivers. It is therefore apparent that if one posses the skills to design filters yielding conventional acoustic HP and LP transfer functions, such as Linkwitz-Riley, Bessel or Butterworth, then the design of the required HP and LP sections of the present crossover should present no more of a challenge. Care must be taken, just as with the use of conventional filters, to assure that the acoustic centers of the drivers are correctly aligned. Otherwise, the summed driver response will not exhibit the correct amplitude and phase response.

Development of the passive equalization circuit, however, can be a complex task. Ideally, the impedance response for the summed network must be compensated to yield, as close as possible, constant impedance. While not always a simple task, this can generally be done through the addition of various Zobels and RLC shunts across the speaker terminals. Due to the double peak nature of the woofer impedance in vented boxes, impedance compensation for such systems may prove difficult. For that reason, the present approach of passive equalization is again found to be better suited for sealed box systems. In the case of significant tweeter/woofer impedance miss matches, it is also possible to compensate the tweeter and woofer networks separately, and then develop separate equalization circuits for each. Which ever approach is taken, here is another area in which the power of optimization programs such as CALSOD can be brought to bare. With the filter components known, they can be held fixed and the full power of the optimization procedure applied to the equalization circuit. Of course, it is always possible of circumvent the complexity of passive equalization and default to an active equalization circuit, the design of which reduces to not much more than a text book exercise.

An additional and important note regarding the use of passive equalization is that the series attenuation resistor in the equalization circuit must be accounted for in the design of the woofer enclosure. This resistor will result in an increase in the value of Qes of the driver, and will be reflected in an increased value for Qts. The net result will be the need for a greater box volume for correct woofer alignment. And while there is the loss in sensitivity due to the presents of this resistor, the aforementioned effects on the driver parameters will result in a lower system f_3 for a correctly aligned woofer.

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